D 10667 **(Pages : 2)** **Name.......................................**

Reg. No..................................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS–UG)

Mathematics

MTS 5B 06—BASIC ANALYSIS

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Is the union of two disjoint denumerable sets denumerable ?
2. If $a, b \in \mathbb{R}$ with $ab = 0$, then prove that either $a = 0$ or $b = 0$.
3. If $a \in \mathbb{R}$ is such that $0 \leq a < \varepsilon$ for every $\varepsilon > 0$, then show that $a = 0$.
4. Find all real numbers x satisfying the inequality $x^2 > 3x + 4$.
5. If $0 < c < 1$, then show that $0 < c^2 < c < 1$.
6. If x and $y \in \mathbb{R}$ with $x < y$ prove that there exists an irrational number z such that $x < z < y$.
7. State characterization theorem for intervals.
8. Test the convergence of $\left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \ \ldots\right)$.
9. Show that every convergent sequence is a Cauchy sequence.
10. Define Supremum of a set and give example of a set which has no Supremum.
11. What can be said about the complex number z if $z = -\overline{z}$.
12. Find modulus of the complex number $z = -9i$.
13. Find real and imaginary parts of the complex function $f(z) = \overline{z}$ as functions of r and θ.
14. State nested interval property.
15. Write the equation of (a) a closed disk of radius ρ centred at z_o ; (b) equation of a circle with centre z_o and radius ρ.

(10 × 3 = 30 marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. State and prove Cantor's theorem.

17. Prove that there does not exist a rational number r such that $r^2 = 2$.

18. Solve the inequality $|2x - 1| \leq x + 1$.

19. Let S be a non-empty set in $\mathbb{R}$, that is bounded above. Prove that Sup $(a + S) = a +$ Sup S.

20. State and prove Archimedean property.

21. Prove that a sequence in $\mathbb{R}$ can have atmost one limit.

22. Find the image of the half plane Re $z \geq 2$ under the mapping W = iZ.

23. Prove that $|z_1 - z_2| \geq \big||z_1| - |z_2|\big|$.

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) State and prove Arithmetic-geometric inequality.

    (b) Let $a, b, c \in \mathbb{R}$. Then if $ab < 0$ then show that either $a > 0$ and $b < 0$ or $a < 0$ and $b > 0$.

    (c) If 1 < C, then show that $1 < C < C^2$.

25. (a) Prove that every contractive sequence is a Cauchy sequence.

    (b) Prove that if a sequence X of real numbers converges to a real number x, then any subsequence of X also converge to x.

26. (a) The polynomial equation $x^3 - 7x + 2 = 0$ has a solution between 0 and 1. Use an approximate contractive sequence to calculate the solution correct to 4 decimal places.

    (b) Show that $\lim \left(n^{\frac{1}{n}} \right) = 1$.

27. (a) Find an upperbound for $\left| \dfrac{1}{z^4 - 5z + 1} \right|$ if $|z| = 2$.

    (b) Find the image of the vertical strip $2 \leq \text{Re}\, Z < 3$ under the mapping $f(Z) = 3Z$.

    (c) Find the domain of $f(z) = \dfrac{iz}{|z| - 1}$.

(2 × 10 = 20 marks)