

C 20648

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Name.....

Reg. No.....

**SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022**

(CBCSS-UG)

Mathematics

MTS 6B 13—DIFFERENTIAL EQUATIONS

(2019 Admissions)

Time : Two Hours and a Half

Maximum : 80 Marks

**Section A***Answer at least ten questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 30.*

- Find the general solution of the differential equation  $\frac{dy}{dt} = -ay + b$  where a,b are positive real numbers.
- Determine the values of  $r$  for which  $e^{rt}$  is a solution of the differential equation  $y''' - 3y'' + 2y' = 0$ .
- Using method of integrating factors solve the differential equation  $\frac{dy}{dt} - 2y = 4 - t$ .
- Find the solution of the differential equation :

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}, y(0) = -1.$$

- Find the Wronskian of the functions  $\cos^2 \theta, 1 + \cos(2\theta)$ .
- Find the general solution of the differential equation  $y'' + 2y' + 2y = 0$ .
- Let  $y = \phi(x)$  be a solution of the initial value problem :

$$(1+x^2)y'' + 2xy' + 4x^2y = 0, y(0) = 0, y'(0) = 1.$$

Determine  $\phi'''(0)$ .

- Determine a lower bound for the radius of convergence of series solutions about each given point  $x_0 = 4$  for the given differential equation  $y'' + 4y' + 6xy = 0$ .

**Turn over**

9. Find the Laplace transform of the function  $\sin(at)$ .
10. Find the inverse Laplace transform of  $\frac{n!}{(s-a)^{n+1}}$  where  $s > a$ .
11. Let  $u_c(t)$  be unit step function and  $L(f(t)) = F(s)$ . Show that :
- $$L(u_c(t)f(t-c)) = e^{cs}F(s).$$
12. Find the inverse Laplace transform of the following function by using the convolution theorem
- $$\frac{1}{s^4(s^2+1)}.$$
13. Solve the boundary value problem :
- $$y'' + y = 0, y(0) = 0, y(\pi) = 0.$$
14. Define an even function and show that if  $f(x)$  is an even function then :
- $$\int_{-L}^L f(x) dx = 2 \int_0^L f(x) dx.$$
15. Define the following partial differential equations :
- heat conduction equation.
  - one-dimensional wave equation.

(10 × 3 = 30 marks)

**Section B***Answer at least five questions.**Each question carries 6 marks.**All questions can be attended.**Overall Ceiling 30.*

16. Let  $y_1(t)$  be a solution of  $y' + p(t)y = 0$  and let  $y_2(t)$  be a solution of  $y' + p(t)y = g(t)$ .  
Show that  $y(t) = y_1(t) + y_2(t)$  is also a solution of equation  $y' + p(t)y = g(t)$ .
17. Find the value of  $b$  for which the following equation is exact, and then solve it using that value of  $b$ .
- $$(xy^2 + bx^2y) + (x+y)x^2y' = 0.$$
18. Solve the initial value problem
- $$y'' + 4y = t^2 + 3e^t, y(0) = 0, y'(0) = 2.$$

19. Use method of variation of parameters find the general solution of :

$$y'' + 4y = 8 \tan t, -\pi/2 < t < \pi/2.$$

20. Find the solution of the initial value problem :

$$2y'' + y' + 2y = \delta(t-5), y(0) = 0, y'(0) = 0.$$

here  $\delta(t)$  denote the unit impulse function.

21. Using Laplace transform solve the initial value problem :

$$y'' + 4y = 0, y(0) = 3, y'(0) = -1.$$

22. Find the co-efficients in the Fourier series for  $f$ :

$$f(x) = \begin{cases} 0, & -3 < x < -1 \\ 1, & -1 < x < 1 \\ 0, & 1 < x < 3 \end{cases}$$

Also suppose that  $f(x+6) = f(x)$ .

23. Find the solution of the following heat conduction problem :

$$100u_{xx} = u_t, 0 < x < 1, t > 0$$

$$u(0, t) = 0, u(1, t) = 0, t > 0$$

$$u(x, 0) = \sin(2\pi x) - \sin(5\pi x), 0 \leq x \leq 1.$$

(5 × 6 = 30 marks)

### Section C

*Answer any two questions.  
Each question carries 10 marks.*

24. Find the general solution of the following differential equation using the method of integrating factors :

$$\frac{dy}{dt} + \frac{1}{2}y = \frac{1}{2}e^{t/3}.$$

Draw some representative integral curves of the differential equation and also find the particular solution whose graph contains the point (0,1).

25. Find a series solution of the differential equation :

$$y'' + y = 0, -\infty < x < \infty.$$

26. Find the Laplace transform of  $\int_0^t \sin(t-\tau)\cos\tau \, d\tau$

27. Find the temperature  $u(x, t)$  at any time in a metal rod 50 cm long, insulated on the sides, which initially has a uniform temperature of 20°C throughout and whose ends are maintained at 0°C for all  $t > 0$ .

(2 × 10 = 20 marks)