C 20212 (Pages : 3) Name.....

Reg. No.....

SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 13 (E02)—LINEAR PROGRAMMING

(2014 to 2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Define vertex of a convex set.
2. Find the convex hull of $\{x_1, x_2\} \subset \mathbb{R}^2$.
3. Define objective function.
4. While solving an LPP by graphical method, when does one conclude that there exist infinitely many points in the feasible region at which the objective function attains optimum ?
5. Define a slack variable.
6. Write the matrix form of general LPP.
7. What is meant by feasible solution to an LPP ?
8. What is meant by optimal solution to an LPP ?
9. When do you say that a transportation problem is unbalanced ?
10. State the necessary and sufficient condition for the existence of a feasible solution to general transportation problem.
11. Write the number of basic variables of the general transportation problem at any stage of feasible solution.
12. In assignment problem, what is the value of decision variable ?

(12 × 1 = 12 marks)

Section B

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Prove that the intersection of two convex sets is a convex set.
14. Write the standard form of LPP.
15. What is the basic principle of linear programming ?

Turn over

16. Does a feasible solution exist for the LPP : Maximize $z = 2x_1 + 10x_2$ subject to $x_1 - x_2 \geq 1, -x_1 + x_2 \geq 2, x_1 \geq 0, x_2 \geq 0$? Give reasons.
17. For linear inequalities, show that the solution set for a group of inequalities is a convex set.
18. Write the dual of the LPP :

    Maximize $f(x) = 3x_1 + 2x_2$ subject to $2x_1 + x_2 \leq 20, x_1 + 3x_2 \leq 20, x_1 \geq 0, x_2 \geq 0$.
19. Define a loop in transportation table.
20. How does a loop in a transportation table related to a basic feasible solution ?
21. Define a triangular basis. Write the role of triangular basis in transportation problem.
22. How do you solve an unbalanced transportation problem ?
23. Write the disadvantage of North-West corner rule.
24. "An assignment problem is a particular case of a transportation problem." Justify.

(9 × 2 = 18 marks)

Section C

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Show that the set $S = \{(x_1, x_2) : 5x_1 + 2x_2 \geq 10, 2x_1 + 5x_2 \geq 10\}$ is convex.
26. (a) Define convex linear combination of a finite set of vectors.
    (b) Show that the set of all convex linear combinations of a finite number of vectors $u_1, u_2, u_3, \ldots, u_k \in \mathbb{R}^n$ is a convex set.
27. Prove that the set of feasible solutions to an LPP is a convex set.
28. Show that the following system of linear equations has a degenerate solution :

    $2x_1 + x_2 - x_3 = 2, 3x_1 + 2x_2 + x_3 = 3$.
29. Let $x_1 = 2, x_2 = 4$ and $x_3 = 1$ be a feasible solution to the system of equations $2x_1 - x_2 + 2x_3 = 2$, $x_1 + 4x_2 = 18$. Reduce this feasible solution to a basic feasible solution.
30. Use graphical method to solve the following LPP :

    Maximize $z = 6x_1 + x_2$ subject to $2x_1 + x_2 \geq 3, x_2 - x_1 \geq 0, x_1 \geq 0, x_2 \geq 0$.
31. Prove that the dual of the dual is the primal.
32. How do you resolve the problem of degeneracy in transportation problem ?

33. Obtain an initial basic feasible solution to the following transportation problem using North-West Corner rule :

	D_1	D_2	D_3	D_4	Supply
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
Demand	16	18	31	25	

(6 × 5 = 30 marks)

Section D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Use Simplex method to solve the LPP :

Maximize $Z = 4x_1 + 10x_2$ subject to

$2x_1 + x_2 \leq 50$

$2x_1 + 5x_2 \leq 100$

$2x_1 + 3x_2 \leq 90$ and $x_1 \geq 0, x_2 \geq 0$.

35. Use Vogel's approximation method to obtain an initial basic feasible solution to the transportation problem :

	D_1	D_2	D_3	D_4	Availability
O_1	11	13	17	14	250
O_2	16	18	14	10	300
O_3	21	24	13	10	400
Demand	200	225	275	250	

36. A department head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimate of the time each man would take to perform each task is given in the matrix below :

Tasks	Men			
	T_1	T_2	T_3	T_4
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How the task should be allocated, one task to a man, so as to minimize the total man-hours ?

(2 × 10 = 20 marks)