D 50672 **(Pages : 4)** **Name.......................................**

Reg. No...................................

FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2023

Mathematics

MTS 5D 03—LINEAR MATHEMATICAL MODELS

(2020 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 20.

1. Find the equation of the line through $(4, 2)$ and $(1, 3)$ in slope intercept form.

2. Find k so that the line through $(4, -1)$ and $(k, 2)$ is perpendicular to $5x - 2y = -1$.

3. Solve the system of equations using echelon method :

$$\frac{x}{2} + y = \frac{3}{2}$$
$$\frac{x}{3} + y = \frac{1}{3}.$$

4. Let $A = \begin{bmatrix} -2 & 4 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & 2 \\ 4 & 0 \end{bmatrix}$. Find 7B – 3A.

5. Find the values of variables in the matrix equation $\begin{bmatrix} 2 & x \\ y & 6 \\ 5 & z \end{bmatrix} = \begin{bmatrix} a & -1 \\ 4 & 6 \\ p & 7 \end{bmatrix}$.

6. Graph the linear inequality $2x - 3y \leq 12$.

Turn over

7. Graph the feasible region for the following system of inequalities and tell whether the region is bounded or unbounded :

$$x + y \le 1$$
$$x - y \ge 2.$$

8. Identify all variables used and express the statement given below as linear inequalities :

Product A requires 3 hours on a machine, while product B needs 5 hours on the same machine. The machine is available for at most 60 hours per week.

9. Use the graphical method to solve the following linear programming problem :

$$\begin{aligned} &\text{Maximize } z = 2x + 4y \\ &\text{subject to } \quad 3x + 2y \le 12 \\ &\qquad\qquad 5x + y \ge 5 \\ &\qquad\qquad x \ge 0, y \ge 0. \end{aligned}$$

10. Set up the initial tableau for the following linear programming problem :

$$\begin{aligned} &\text{Maximize } z = 120x_1 + 40x_2 + 60x_3 \\ &\text{subject to } \quad x_1 + x_2 + x_3 \le 100 \\ &\qquad\qquad 10x_1 + 4x_2 + 7x_3 \le 500 \\ &\qquad \text{with } x_1 \ge 0, x_2 \ge 0, x_3 \ge 0. \end{aligned}$$

11. Write the solution that can be read from the following simplex tableau :

x_1	x_2	x_3	s_1	s_2	z	
1	0	4	5	1	0	8
3	1	1	2	0	0	4
−2	0	2	3	0	1	28

12. Explain standard minimum form of a linear programming problem.

Section B

Answer any number of questions.
Each question carries 5 marks.
Ceiling is 30.

13. The sales of a small company were $27,000 in its second year of operation and $63,000 in its fifth year. Let y represent sales in the x^{th} year of operation. Assume that the data can be approximated by a straight line.

    a) Find the slope of the sales line, and give an equation for the line in the form $y = mx + b$.

    b) Use your answer from part (a) to find out how many years must pass before the sales surpass $1,00,000.

14. Consider the following table of data :

x	:	1	1	2	2	9
y	:	1	2	1	2	9

Calculate the least square line and the correlation co-efficient.

15. Explain briefly Gauss-Jordan method to solve the linear system.

16. Find the inverse of $A = \begin{bmatrix} -1 & -1 & -1 \\ 4 & 5 & 0 \\ 0 & 1 & -3 \end{bmatrix}$.

17. Using the open model, find the production matrix for the input - output matrix $A = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$

and demand matrix $D = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

18. Introduce slack variables as necessary and write the initial simplex tableau for the linear programming problem given below :

Find $x_1 \geq 0$ and $x_2 \geq 0$ such that
$4x_1 + 2x_2 \leq 5$
$x_1 + 2x_2 \leq 4$
and $z = 7x_1 + x_2$ is maximized.

Turn over

19. Pivot about indicated 2 of the following initial simplex tableau :

x_1	x_2	x_3	s_1	s_2	z	
1	2	4	1	0	0	56
2	$\boxed{2}$	1	0	1	0	40
-1	-3	-2	0	0	1	0

Section C

Answer any **one** *question.*
The question carries 10 *marks.*

20. Solve the following system of equations by finding inverse of the co-efficient matrix by Gauss - Jordan method :

$$x + 3y - 2z = 4$$
$$2x + 7y - 3z = 8$$
$$3x + 8y - 5z = -4.$$

21. Find the dual of the following linear programming problem and solve using simplex method :

Minimize $w = 3y_1 + 2y_2$
subject to $y_1 + 3y_2 \geq 6$
$2y_1 + y_2 \geq 3$
with $y_1 \geq 0, y_2 \geq 0.$

(1 × 10 = 10 marks)