D 50656 **(Pages : 3)** **Name.......................................**

Reg. No..................................

FIFTH SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2023

Mathematics

MTS 5B 05—THEORY OF EQUATIONS AND ABSTRACT ALGEBRA

(2019 Admissions only)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 25.

1. Check whether $x^5 - 3x^4 + x^2 - 2x - 3$ is divisible by $x - 3$.

2. Expand the polynomial $4x^3 - 7x^2 + 5x + 3$ in powers of $x + 2$.

3. Find the polynomial of lowest degree that vanishes at $x = -1, 0, 1$ and takes the value 1 for $x = 2$.

4. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$.

5. Separate the roots of the equation $2x^5 - 5x^4 + 10x^2 - 10x + 1 = 0$.

6. Let n be a positive integer. Prove that the congruence class $[a]_n$ has a multiplicative inverse in Z_n iff $(a, n) = 1$.

7. Find the multiplicative order of $[2]$ and $[5]$ *in* $\mathbb{Z}_{17}^{X}$.

8. Check whether the relation ~ on R defined by a~b if $a \le b$ is an equivalence relation.

9. Let S be any set .If σ and τ are disjoint cycles in Sym(S) then $\sigma\tau = \tau\sigma$. Prove it.

10. Find the order of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 6 & 7 & 5 & 1 & 8 & 2 & 3 \end{pmatrix}$.

11. Let G be a group and $a, b, c \in G$. Prove that if $ab = ac$ then $b = c$.

Turn over

12. Show that if G is a finite group with an even number of elements then there must exist an element $a \in G$ with $a \neq e$ such that $a^2 = e$.

13. Is S_3 cyclic ? Justify your answer.

14. Give the subgroup diagram of $\mathbb{Z}_{24}$.

15. Check whether the set $\{m + n\sqrt{2} \mid m, n \in \mathbb{Z} \text{ and } n \text{ is even}\}$ is a subring of the field of real numbers.

(Ceiling 25)

Section B

Answer any number of questions.
Each question carries 5 marks.
Ceiling is 35.

16. Factorise the polynomial $x^6 - 1$ into linear factors.

17. Find the limits of the moduli of roots of the equation $2x^6 - 7x^5 - 10x^4 + 30x^3 - 60x^2 + 10x - 50 = 0$.

18. Examine for integral roots $x^4 + 8x^3 - 7x^2 - 49x + 76 = 0$.

19. If G is a group and $a, b \in G$, then prove that each of the equations $ax = b$ and $xa = b$ has a unique solution.

20. Write $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 4 & 10 & 5 & 7 & 8 & 2 & 6 & 9 & 1 \end{pmatrix}$ as Product of disjoint cycles. Find its order and inverse.

21. Find all cyclic subgroups of $\mathbb{Z}_6$.

22. Let G be a group and H and K be subgroups of G.If $h^{-1}kh \in K$ for all $h \in H$ and $k \in K$ then prove that HK is a subgroup of G.

23. Find the cyclic subgroup generated by $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ in $GL_2(\mathbb{Z}_3)$.

(35 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum marks 20.

24. Solve the cubic equation : $x^3 + 9x - 6 = 0$.

25. If m and n are positive integers such that gcd(m, n) = 1 then prove that $\mathbb{Z}_{mn} \cong \mathbb{Z}_m \times \mathbb{Z}_n$.

26. State and prove Cayley's theorem.

27. Let G be a group. Then show that Aut(G) is a group and Inn(G) is a normal subgroup of Aut(G).

(2 × 10 = 20 marks)