D 10666 **(Pages : 3)** **Name.......................................**

Reg. No..................................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5B 05—THEORY OF EQUATIONS AND ABSTRACT ALGEBRA

(2019 Admissions)

Time : Two Hours and a Half — Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Show that $x^5 - 3x^4 + x^2 - 2x - 3$ is divisible by $x - 3$.

2. Factorize into linear factors the polynomial $x^4 - 1$.

3. Write a cubic equation with roots $1,\ 1+i,\ 1-i$.

4. State Identity theorem.

5. How many real roots has the equation $x^4 - 4ax + b = 0$.

6. Make addition and multiplication tables for $\mathbb{Z}_2$.

7. Check whether the relation on $\mathbb{R}$ defined by $a \sim b$ if $a - b \in \mathbb{Q}$ is an equivalence relation.

8. Consider the permutations $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$. Compute $\sigma\tau$ and $\tau\sigma$.

9. Let G be a group and $a, b \in G$. Show that $(ab)^{-1} = b^{-1}a^{-1}$.

10. Write a subgroup of $(\mathbb{Z}, +)$.

Turn over

11. Check whether $\mathbb{Z} \times \mathbb{Z}$ is cyclic.

12. Find order of the permutation $(1, 3)(2, 6)(1, 4, 5)$.

13. Let $\Phi : G_1 \to G_2$ be a group homomorphism. Show that $\Phi(e) = e'$ where e and e' are identity elements of G_1 and G_2 respectively.

14. Define a Ring.

15. Give example of an integral domain.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Solve $x^5 - 3x^4 + 4x^3 - 4x + 4$ having the root $1 + i$.

17. Solve the cubic equation $2x^3 - x^2 - 18x + 9 = 0$ whose roots are a, b, c with $a + b = 0$.

18. Find an upper limit of the positive roots of the equation $2x^5 - 7x^4 - 5x^3 + 6x^2 + 3x - 10 = 0$.

19. Prove that set of all even permutations of S_n is a subgroup of S_n.

20. Define * on $\mathbb{Z}$ by $a * b = a - b$. Check whether $(\mathbb{Z}, *)$ is a group.

21. Check whether $\mathbb{Z}_n$ is cyclic.

22. Draw the subgroup diagram of $\mathbb{Z}_{36}$.

23. Let G_1 and G_2 be groups and let $\Phi : G_1 \to G_2$ be a function such that $\Phi(a\,b) = \Phi(a)\,\Phi(b)$ for all $a, b \in G$. Prove that Φ is $1 - 1$ if and only if $\Phi(x) = e$ implies that $x = e$ for all $x \in G_1$.

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. Examine whether $x^4 - x^3 - x^2 + 19x - 42 = 0$ has integral roots or not.

25. Solve $x^3 - 6x - 6 = 0$ by Cardan's method.

26. Let G be a cyclic group. Show that :

    (a) If G is infinite then $G \cong \mathbb{Z}$.

    (b) If $|G| = n$, then $G \cong \mathbb{Z}_n$.

27. State and prove Lagrange's theorem.

(2 × 10 = 20 marks)